On some fully nonlinear Allen-Cahn type equation

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Abstract

This talk is concerned with a variant of the Allen-Cahn equation,

$$u_t = \big(\Delta u - W'(u)\big)_+, \quad x \in \Omega, \quad t > 0$$

where $(\,\cdot\,)_+$ is the positive-part function, $W(\,\cdot\,)$ is a double-well potential and Ω is a smooth bounded domain of $\mathbb{R}^N$. Then time-evolution of $u(x,t)$ is constrained to be non-decreasing. Such a unidirectional evolution appears in the study of Damage Mechanics, where $u(x,t)$ is supposed to be a phase parameter corresponding to the extent of damage in some material (hence its evolution is naturally supposed to be monotone, and moreover, the evolution is often described by a sort of gradient flow). Although the equation above is classified as a fully nonlinear parabolic equation, one can also reduce it to a sort of doubly nonlinear evolution equation, which is more suitable for energy techniques. Due to the constraint on evolution of solutions arising from the positive-part function, each initial data will play a role of an obstacle function from below. Main purpose of this talk is to reveal the long-time behavior of each solution for the Cauchy-Dirichlet problem and to discuss Lyapunov stability for a certain class of equilibria. Particularly, in stability analysis, one faces a difficulty, for equilibria may form a continuum (in a proper energy space), and moreover, another difficulty resides in the non-smooth nature of the constraint problem. To overcome such difficulties, we shall employ the monotonicity of evolution as well as a stability of solutions to elliptic obstacle problems with respect to obstacle functions. This talk is based on a joint work with Messoud Efendiev (München).